THE OPTIMAL LOCATION OF TWO RECYCLING CENTERS

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ABSTRACT. Suppose a municipality optimally locates two recycling/sorting centers to minimize the sum of the transportation costs from i) households to the recycling centers and ii) recycling centers to the landfill. Assume that all household waste is taken to a recycling center, sorted, and the non-recyclables are subsequently transported to the landfill. The landfill location and the proportion of waste recycled are exogenous. The paper shows that the location of the recycling centers is chosen to equalize the marginal costs of the two transportation stages, unless an endpoint condition obtains, in which case one recycling center is located at the landfill. (R1: General Spatial Economics, H7: Publicly Provided Goods)

This paper considers the locational aspects of the optimal design of a municipal waste management program with two recycling centers and a single existing landfill. As transportation costs are a significant part of the overall waste management costs, the paper will suppose that the locations of the recycling centers are chosen to minimize transportation costs. Specifically, it is assumed that all household waste is collected and taken to a recycling center where the recyclables are separated from the non-recyclables. The recyclables are sold off to private recyclers and the non-recyclables are taken to the landfill (whose location is fixed). Thus the problem for the municipality is to choose the locations of the recycling centers to minimize the sum of the costs of the two stages of transportation: 1) from households to recycling centers and 2) from recycling centers to landfill, where the latter cost is appropriately weighted for the reduction in amount of material to be transported. The paper shows that the location of the recycling centers is chosen to equalize the marginal costs of these two transportation stages, unless an endpoint condition[2] obtains, in which case one recycling center is located at the landfill.

I. <u>THE BACKGROUND OF THE PROBLEM</u>

The history of location problems similar to the one of this paper is quite rich. Hotelling's 1929 paper can be taken as the beginning of this literature; it analyzes a special case of the problem of the present paper. Suppose that all waste is recycled, so that the transportation problem simplifies to a single stage problem – since there is nothing to transport from the recycling centers to the landfill. As Hotelling notes, the city should be divided into two regions in such a way that the consumers at the dividing point are indifferent between recycling centers, and within each region, the recycling centers are optimally located at the median of the region.[3] The literature in addition to Hotelling that considers single stage transportation problems is quite large; among the many papers which could be cited are Ye and Yezer (1992) and Drezner (1986). In order to insure that our two-stage problem does not collapse to a single stage problem, it will be assumed throughout that not all waste is recycled.

After Hotelling's seminal paper, it would be many years before two stage transportation problems received adequate attention. Although not phrased as such, a general two-stage model has been constructed by Weslowsky and Love (1971). Their problem is to find the optimal location of facilities that minimizes the weighted sum of distances to (i) two-dimensional regions with uniform distributions (“area destinations”) and (ii) a finite collection of sites with fixed locations (“point destinations”). The area destinations can be thought of as distribution regions, the fixed sites can be thought of as shipping facilities; the objective is to locate transshipment points. (In the language of our model, the distribution region is the city and the fixed site is the landfill and a transshipment point is our recycling center.) The 1971 paper emphases the construction of algorithmic solutions to the optimization problem in the context of several examples of locating one or two facilities. Although we consider only a one-dimensional region, the present paper can be viewed in part as a generalization of Weslowsky and Love's work in that the distribution on the region is allowed to be arbitrary. In addition, endpoint solutions (defined in note 2) are possible here but are not germane in Weslowsky and

Love. Finally, the other contribution here is to determine the region that is served by each of the recycling centers and then to draw some economic conclusions.

Another paper that contains a two-stage model is the recent paper of Braid (1996). In this paper, the objective is to find the optimal location of either two branch facilities or a main facility and a branch so as to minimize distance traveled along a linear city. The two-stage feature results from the response of a decision maker to the state of the branch facilities. For example, the branch facility may represent a branch library. The patron finds the desired material with a given probability at the branch library. If unsuccessful, the second-stage of the trip takes the patron from the branch to the main facility. In our model, some of the municipal waste ends its trip at a recycling center (the branch) while the rest goes to the landfill (the "main" facility). Braid's work is a generalization of Hotelling's work to two locations. Our work generalizes Braid, in that the distribution of the city is of allowed to be arbitrary rather than uniform. Our techniques are general enough to extend to more than two locations and we find general conditions that describe the optimal solution.

The specific location model being generalized in this paper is drawn from Highfill, McAsey, and Weinstein (1994). In that model waste is transported from households to a single recycling center, and then the unrecycled waste is transported to the landfill. The city in the 1994 model is a two-dimensional rectangle with the landfill fixed at the origin. The transportation costs corresponding to the two stages are first from an area to a single point (the city to the recycling center) and second from point to point (the recycling center to the landfill). The former is a U-shaped function whose minimum is at the median of the city, the later is the absolute value function whose minimum is at the landfill. The location of the landfill is the choice variable. Households are distributed uniformly over the city. In that paper an endpoint solution is possible only in the uninteresting case when no recycling occurs. Otherwise, the optimal location for the landfill is the point that equates the marginal reduction in the cost of transporting waste between the households and the recycling center with the marginal cost of transporting non-recyclables between the recycling center and the landfill. The present paper generalizes location model of Highfill, McAsey, and Weinstein (1994) by assuming that the city is finding the optimal location of two recycling centers, and the density of households is not necessarily uniform. Because of the complexities introduced by these assumptions, the city is now assumed to exist along a line rather than in a plane.

While the present paper is primarily a contribution to the location literature, its original problem is motivated by a paper in the literature on municipal recycling, and thus this paper can be seen as a contribution, however limited, to the recycling literature. Such work is relatively new, and has usually focused on an empirical investigation of recycling in specific communities (e.g., Hong, Adams, and Love (1993), Judge and Becker (1993), Reschovsky and Stone (1994), Strathman, Rufolo, and Mildner (1995), Fullerton and Kinnaman (1996)) or a comparison of communities (e.g., Jenkins (1993), Miranda, Everett, Blume, and Roy (1994)). Much of the recent work has been concerned with the pricing strategies for waste management options, e.g., per unit fees for special bags or stickers for household waste. For theoretical work, Morris and Holthausen (1994) and Kinnaman and Fullerton (1995) investigate the effect of fee structures on household recycling. Highfill and McAsey (1997) conduct a theoretical investigation of the dynamics of landfill exhaustion and recycling.

II. THE BASIC MODEL

Suppose a city (or municipality) wishes to establish two recycling centers at the locations which minimize the total transportation cost to the city of hauling residential waste. Assume that the waste generated by the households is distributed spatially according to a (non-negative, bounded, measurable) density function[4] $\rho(x)$ on $\mathbb{R}$. Without loss of generality, we assume that the total amount of waste is 1:

$$\int_{\mathbb{R}} \rho(x)\, dx = 1$$

and also assume that $\rho(x) > 0$ only on some bounded subset of the line.

Let the location of the landfill[5] be fixed at z_L. The city wishes to choose the locations for the recycling centers $z_1, z_2, (z_1 < z_2)$ and a dividing point m to minimize transportation costs. The dividing point m divides the city into two segments; households to the left of m will have their waste taken to the recycling center located at z_1, while those to the right of m will have their waste taken to the recycling center located at z_2. At the recycling centers the waste is sorted into recyclables and nonrecyclables. Denote the exogenous proportion of waste recycled by γ.[6] (Once sorted, the recyclable waste is no longer of concern to the city because the city has contracted with a commercial recycler for its removal.) The remaining proportion of waste, $1 - \gamma$, is transported to the landfill. The distance between any two arbitrary points x_1 and x_2 is $|x_1 - x_2|$. Thus, a weighted distance function can be defined:

$$\phi(x, z_i, z_L) = |x - z_i| + (1 - \gamma)\, |z_i - z_L| \;\; i \,=\, 1,\, 2. \tag{1}$$

For either recycling center, the first term of this function is the distance between the households and the recycling center while the second term is the distance between the recycling center and the landfill multiplied by the proportion of the waste which is taken to the landfill, namely, $(1 - \gamma)$.

In sum, the total transportation cost is

$$F(z_1, z_2, m) = \int_{-\infty}^{m} \rho(x)\phi(x, z_1, z_L)\, dx + \int_{m}^{\infty} \rho(x)\phi(x, z_2, z_L)\, dx \tag{2}$$

The necessary conditions for the problem follow. Let (z_1, z_2, m) give the minimum of the objective function. The partial derivatives used in finding the first order conditions are:

$$\frac{\partial F}{\partial z_1} = \int_{-\infty}^{m} \rho(x)(\text{sgn}(z_1 - x) + (1-\gamma)\text{sgn}(z_1 - z_L))\,dx,$$

$$\frac{\partial F}{\partial z_2} = \int_{m}^{\infty} \rho(x)(\text{sgn}(z_2 - x) + (1-\gamma)\text{sgn}(z_2 - z_L))\,dx,$$

and

$$\frac{\partial F}{\partial m} = \rho(m)\phi(m, z_1, z_L) - \rho(m)\phi(m, z_2, z_L)$$

where $\text{sgn}(y) = \begin{cases} 1 & \text{if } y > 0 \\ 0 & \text{if } y = 0. \\ -1 & \text{if } y < 0 \end{cases}$ The resulting first order conditions are summarized in the following conditions.

Note that the absolute value function in $\phi(x, z_{\text{i}}, z_L)$ implies that there are five separate cases. In all cases recall that $z_1 < z_2$. The cases are numbered by where the landfill is located relative to z_1 and z_2, starting from a landfill located to the right of the city: $z_L > z_2$.

Case I: $z_L > z_2$

$$m = \frac{2-\gamma}{2} z_1 + \frac{\gamma}{2} z_2 \tag{3}$$

$$\int_{z_1}^{m} \rho(x)\,dx = \frac{\gamma}{2}\int_{-\infty}^{m} \rho(x)\,dx \quad , \quad \int_{z_2}^{\infty} \rho(x)\,dx = \frac{\gamma}{2}\int_{m}^{\infty} \rho(x)\,dx$$

Case II: $z_L = z_2$

$$m = \frac{2-\gamma}{2} z_1 + \frac{\gamma}{2} z_2 \tag{4}$$

$$\int_{z_1}^{m} \rho(x)\,dx = \frac{\gamma}{2}\int_{-\infty}^{m} \rho(x)\,dx$$

Case III: $z_1 < z_L < z_2$

$$m = \frac{z_1 + z_2}{2} - (1-\gamma)\left(z_L - \frac{z_2 + z_1}{2}\right) \tag{5}$$

$$\int_{z_1}^{m} \rho(x)\,dx = \frac{\gamma}{2}\int_{-\infty}^{m} \rho(x)\,dx\,, \quad \int_{m}^{z_2} \rho(x)\,dx = \frac{\gamma}{2}\int_{m}^{\infty} \rho(x)\,dx$$

Case IV: $z_L = z_1$

$$m = \frac{\gamma}{2}\,z_1 + \frac{2-\gamma}{2} z_2 \tag{6}$$

$$\int_{m}^{z_2} \rho(x)\,dx = \frac{\gamma}{2}\int_{m}^{\infty} \rho(x)\,dx$$

Case V: $z_L < z_1$

$$m = \frac{\gamma}{2}\,z_1 + \frac{2-\gamma}{2} z_2 \tag{7}$$

$$\int_{-\infty}^{z_1} \rho(x)\,dx = \frac{\gamma}{2}\int_{-\infty}^{m} \rho(x)\,dx\,, \quad \int_{m}^{z_2} \rho(x)\,dx = \frac{\gamma}{2}\int_{m}^{\infty} \rho(x)\,dx$$

III. THE OPTIMAL LOCATIONS OF THE RECYCLING CENTERS

The preceding section provides an analytical description of the optimal location of the recycling centers. In each of the cases, the equations define a critical point, which then must be checked to see if it gives a minimum total cost. This section gives the intuition of these locations. First note that the city is divided at the point m. This point is chosen so that the households located at m are indifferent between having their waste taken to the left or right recycling center. Although the location of the dividing point depends on the location of the recycling centers, once it is known, the problem reduces to two (symmetric) problems of choosing a single location in a fixed region. Because of this symmetry we will discuss only the right side of the city ($z > m$). As is made clear from the first order conditions, several cases result, but from the reduction to considering only the right side of the city, we need only consider cases I, II and III.

To investigate case I, assume z_L is so large that $m < z_2 < z_L$. Assuming m has been chosen, we need consider only the second integral in (2). Substitute the definition of ϕ

from (1) into this part of (2) gives

$$\int_m^\infty \rho(x)(|x - z_2| + (1-\gamma)|z_2 - z_L|)\,dx.$$

This expression can be split into the sum of two functions $F_1(z_2) + F_2(z_2)$. The first function, $F_1(z_2) = \int_m^\infty \rho(x)|x - z_2|\,dx$, represents the stage 1 cost of transportation from households to a recycling center while the second function, $F_2(z_2) = \left((1-\gamma)\int_m^\infty \rho(x)\,dx\right)|z_2 - z_L|$, represents the stage 2 cost of transportation from recycling center to landfill. The first function $F_1(z_2)$ has a general U-shape (although not necessarily convex) as a function of z_2. The minimum of F_1 is at the median, m_2, of the density ρ when restricted to the interval $[m, \infty)$.[7] The second function, F_2, is a multiple of an absolute value function and is zero at $z_2 = z_L$. These cost functions are shown in Figure 1. The horizontal axis shows the (one-dimensional) city while the vertical axis shows simultaneously the stage 1 cost and the stage 2 cost associated with the right recycling center. Clearly the optimal location for z_2 must be between the median and the landfill location since each of these locations minimizes part of the total cost. So the interval $[m_2, z_L]$ between the landfill and the median forms a natural interval for the optimal location. Using the graph to search for the optimal location, begin at the median and move right. Shifting the location of z_2 to the right from the median will increase the cost of the stage 1 transportation; the marginal cost of such a shift is the slope of the function F_1. The same shift rightward from the median decreases the stage 2 transportation costs; the marginal benefit of the shift is the slope of the absolute value function F_2.[8] The critical point, given algebraically by the first order condition $\frac{\partial F}{\partial z_2} = 0$, is the point at which the marginal benefit equals the marginal cost. This is the location of z_2 between m_2 and z_L so that $F_1'(z_2) = -F_2'(z_2)$ where the two slopes are equal, as shown on Figure 1. We will call this optimal point the cost/benefit point associated with the right recycling region and denote it by c/b_2. Thus $z_2 = c/b_2$ for case I. It is interesting that if the landfill lies far enough to the right so that $c/b_2 < z_L$ and hence case

I obtains, then the location of the landfill does not enter explicitly into the optimal location of z_2. That is, any landfill located further to the right would give the same optimal z_2.

For a second illustration of the intuition behind the optimal location, consider the case that the landfill is left of the interval between the cost/benefit point c/b_2 and the median m_2: $z_L < c/b_2 < m_2$. (This is one of the cases III, IV or V.) Once again the optimal location of the right recycling center is located at the cost/benefit point. Note that the first order conditions again give an analytical description but graphically, the point c/b_2 is found where the tangent to the left of the median m_2 is parallel to the downward sloping part of F_2.

For a final illustration, consider Figure 2 in which the cost/benefit point lies outside of the required interval $[m_2, z_L]$ between the median and the landfill. (This is case II.) Although the first order conditions produce a candidate at c/b_2, this candidate is not viable since $c/b_2 \notin [m_2, z_L]$. The explanation here is that choosing the critical point c/b_2 would require transporting the majority of the waste in the right region past the landfill to c/b_2 and then back to the landfill. The optimal solution in this case is, in optimization language, an end point solution: locate the right recycling center at the landfill so that $z_2 = z_L$.

IV. A CITY WITH A UNIFORM DISTRIBUTION

This section will examine a city whose waste is uniformly distributed in order to provide an example of the general results. While somewhat outside the spirit of a "general density function," this example shows that even with the simplest of density functions, the problem of optimally locating two recycling centers has a complex answer. The specific goal is to do a "comparative statics" of landfill location, that is, to establish the relation between landfill location, level of recycling and the case that obtains. Suppose $\rho(x) = 1$ on $[0, 1]$. The first order conditions can be calculated from (3)-(7) and the following results (derived in the appendix) show the dependence of the location data m, z_1, and z_2 on the exogenous values of the level of recycling γ and location of the landfill z_L.

Case I: $z_L > z_2$

$$m = \frac{1}{2}\,,\ z_1 = \frac{2-\gamma}{4},\ z_2 = \frac{4-\gamma}{4} \tag{3'}$$

Case II: $z_L = z_2$

$$m = \frac{2}{4-\gamma}\,z_L\,,\ z_1 = \frac{2-\gamma}{4-\gamma}\,z_L,\ z_2 = z_L \tag{4'}$$

Case III: $z_1 < z_L < z_2$

$$m = \frac{\gamma^2 - 2\gamma(2z_L+1) + 4z_L}{2(\gamma^2 - 4\gamma + 2)} \tag{5'}$$

$$z_1 = \frac{(2-\gamma)(\gamma^2 - 2\gamma(2z_L+1) + 4z_L)}{4(\gamma^2 - 4\gamma + 2)}$$

$$z_2 = \frac{\gamma(2z_L - 1)}{2(\gamma^2 - 4\gamma + 2)} + \frac{\gamma + 4z_L}{4}$$

Case IV: $z_L = z_1$

$$m = \frac{2z_L + 2 - \gamma}{4-\gamma},\ z_1 = z_L,\ z_2 = \frac{2 + (2-\gamma)z_L}{4-\gamma} \tag{6'}$$

Case V: $z_L < z_1$

$$m = \frac{1}{2},\ z_1 = \frac{\gamma}{4},\ z_2 = \frac{2+\gamma}{4} \tag{7'}$$

Some numerical examples may be useful. Recall that the waste is distributed uniformly on the interval [0,1]. Suppose the landfill is located at .9 and 50% of waste is recycled. In this case, the city is divided into two equal-size recycling regions, i.e., m = .5, and the recycling centers are located at .375 and .875 . This is an example of a Case I solution – which implies that the locations of recycling centers would be the same if the landfill were anywhere to the right of .9 as well. For a second example, suppose the recycling center is at .75 and 50% of waste is recycled. In this case the city is divided at .429. The left recycling center is at .321 and the right recycling center is located at the landfill, making this an example of a Case II solution. Finally, suppose the landfill is located at .55 and 75% of waste is recycled. Then the city divides at .443 into the two recycling regions, and the recycling centers are at .277 and .652. This is an example of a Case III solution.

In order to distinguish between the various cases as in the preceding examples it is helpful to construct a "parameters diagram" (Figure 3). This diagram shows the case that is associated with each ordered pair (z_L, γ). The boundaries between the cases are found by using the information about "adjacent" cases. For example, Case I requires $z_L > z_2$ while Case II requires $z_L = z_2$. So the boundary between these cases is found by setting the expression for z_2 in Case I equal to the value for z_2 in Case II: $\frac{4-\gamma}{4} = z_L$ or $\gamma = 4(1 - z_L)$. (This is the slanted straight line with negative slope on Figure 3.) Similarly the boundary between Cases IV and V is given by $\gamma = 4z_L$. The curve from the origin to the point (0.75,1) is given by $z_L = \frac{\gamma(4-\gamma)}{4}$ while the remaining curve from (1,0) to (.25,1) is given by $z_L = \frac{(\gamma-2)^2}{4}$.

The locations of the recycling centers z_i and the dividing point m can now be obtained from a given landfill location x_L and level of recycling γ: locate the point (x_L, γ) on the parameters diagram to determine the appropriate case; then use the associated first order condition given by one of (3′) $-$ (7′). For example, notice that Case III will obtain only for situations where γ is relatively high (greater than $2 - \sqrt{2}$ $\approx .586$) and the landfill is located at or near the middle of the city. Case I will occur if the landfill is near the right boundary of the city, while Case V will occur if the landfill is near the left boundary of the city. Cases II and IV are the "most likely" cases to occur in the sense that more area in the parameters diagram is made up of these cases than the other cases.

V. CONCLUSION

The model of this paper can be used to optimally locate two recycling centers for any city with a fixed location landfill and an exogenous recycling proportion. Although Figure 3 considers the specific case of a uniform distribution, it can also be used to indicate the kinds of general results available from the model of this paper. Supposing that the most important parameters of a city are its landfill location (z_L) and the proportion of waste which is recycled (γ), Figure 3 indicates which of the Cases of the paper obtains for the various possible combinations of landfill location and recycling proportion. That is, it can be used to determine the basic characteristics of an optimal solution for cities of various types. The figure indicates that if the proportion of waste recycled is quite low, i.e., γ is close to zero, then Case IV and Case II are very often the cases that pertain. In both of these cases one of the recycling centers is located at the landfill while the other is closer to the center of the city. If, on the other hand, the proportion of waste recycled is quite high then Cases I, III, and V usually pertain. These are Cases in which neither recycling center is located at the landfill. Where the recycling centers are in relation to the landfill depends, of course, on the location of the landfill itself. For example, if the landfill is near the center of the city (and most waste is recycled) then Case III obtains and the recycling centers are optimally located so that one is to the left of the landfill and the other is to the right. If the landfill is near the edge of the city, then the recycling centers are both to the left or both to the right of the landfill. Although not immediately apparent from Figure 3, the analysis of the paper indicates that for any combination of landfill location and recycling proportion the recycling centers are always within (or on) the boundaries of the city itself, even if the landfill is not. In another sense however, the location of the recycling center need not change with the location of the landfill. If the landfill is sufficiently far from the city (Cases I and V), the location of the recycling centers will be the same as an identical city with a landfill even further from the city. Finally, the analysis indicates that if any recycling takes place, it is always the case that two recycling centers are better than one given a goal of minimizing transportation costs.

The task of this paper has been to generalize the problem of locating a single landfill to the problem of locating two landfills. The basic method obtained could also be used in the problem of locating three or more landfills $-$ albeit that the number of cases to be considered would be larger. (If there are n recycling centers then there will be $2n + 1$ cases.) Other strategies for generalizing the model might be to construct a more comprehensive "waste management system" model. The present paper considers only the transportation aspects of the optimal locations of recycling centers. But in doing so, of course, it has neglected many important aspects of a waste management. A richer model might incorporate other costs: site acquisition costs, externalities, administrative costs, sorting costs (especially important if the proportion of waste recycled is high). On the other hand, the present model can easily be adapted to several other location problems. Such problems include location of shopping centers in a city, location of main and branch libraries, and location of remote clinics with respect to a main hospital. In each of these problems, the density of the city and the possibility of two-stages of travel will lead to models similar to the present model.

FOOTNOTES

1. Highfill is in the Department of Economics and McAsey and Mou are in the Department of Mathematics of Bradley University, Peoria, Illinois 61625.

2. We will use the term "endpoint solution" to describe solutions for which the usual first order conditions do not give sufficient information for a minimum.

3. While Hotelling only computed his results for a uniform distribution, his observation that the same general results hold for other distributions is certainly true.

4. The density function ρ represents the waste that must be picked up by the city. If the city, for example, requires business to contract separately for their own waste removal, then $\rho(x)$ is zero at such locations.

5. Although the term "landfill" is used, this facility could also be an incinerator or an embarkation point for a community's waste.

6. Municipalities in many states are required to recycle significant proportions (often 25%, occasionally as high as 50% or 75%) of their waste, thus the present paper takes the proportion of waste recycled to be exogenous. Assume the city contracts with a private firm to take away the recyclables for a flat fee, so there are no further variable costs for the recyclables. In all of the analysis of this paper we assume that the transportation costs can be considered independently of any other costs. We also assume that the per unit transportation costs between the households and the recycling center and between the recycling center and the landfill are the same although minor modifications in the model can account for different per unit costs of transportation.

7. To see that F_1 is minimized at the median of ρ on (m, ∞) note that $F_1'(m_2) = \int_m^\infty \rho(x)(\text{sgn}(m_2 - x))\,dx = 0$. This gives $\int_m^{m_2} \rho(x)\,dx = \int_{m_2}^\infty \rho(x)\,dx$. Thus m_2 divides the density on $[m, \infty)$ so that half of the density lies to the left of m_2 and the other to the right. This is the median of the density ρ restricted to $[m, \infty)$.

8. More particularly, it is the positive slope of the absolute value function since $z_2 < z_L$.

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APPENDIX

The derivations of (3′)-(5′) are as follows. The other two cases (6′) and (7′) are similar to (3′)-(4′) and are omitted.

The uniform density is identically one on $[0,1]$. The first order conditions are

$$\frac{\partial F}{\partial z_1} = \int_0^m (\text{sgn}(z_1 - x) + (1-\gamma)\text{sgn}(z_1 - z_L))\, dx = 0,$$

$$\frac{\partial F}{\partial z_2} = \int_m^1 (\text{sgn}(z_2 - x) + (1-\gamma)\text{sgn}(z_2 - z_L))\, dx = 0,$$

and

$$\frac{\partial F}{\partial m} = \phi(m, z_1, z_L) - \phi(m, z_2, z_L) = 0$$

Case I. $z_L > z_2$. In this case, $\text{sgn}(z_i - z_L) = -1$ $(i = 1, 2)$, so the first condition becomes $\int_0^{z_1} dx - \int_{z_1}^m dx = (1-\gamma)m$. This simplifies to $2z_1 - m = (1-\gamma)m$. The second condition becomes $\int_m^{z_2} dx - \int_{z_2}^1 dx = (1-\gamma)(1-m)$. This is the same as $2z_2 - m - 1 = (1-\gamma)(1-m)$. The last condition is $m - z_1 + (1-\gamma)(z_L - z_1) = z_2 - m + (1-\gamma)(z_L - z_2)$. Thus the three first order conditions can be rewritten as $z_1 = (1 - \dfrac{\gamma}{2})m$; $z_2 = 1 - \dfrac{\gamma}{2}(1-m)$; and $m = \dfrac{2-\gamma}{2} z_1 + \dfrac{\gamma}{2} z_2$. Substitute the values of z_1 and z_2 into the remaining equation: $m = (1 - \dfrac{\gamma}{2})^2 m + (\dfrac{\gamma}{2})(1 - \dfrac{\gamma}{2}(1-m))$. Isolating m on the left side of the equation produces $[1 - (1 - \frac{\gamma}{2})^2 - (\frac{\gamma}{2})^2]m = \frac{\gamma}{2}(1 - \frac{\gamma}{2})$. This simplifies to $m = \frac{1}{2}$. Substituting back into the expressions for z_1 and z_2 gives $z_1 = \frac{1}{2}(1 - \frac{\gamma}{2})$ and $z_2 = 1 - \frac{1}{4}\gamma$.

Case II. $z_2 = z_L$. In this case we need only solve for z_1 and m. The first order condition $\frac{\partial F}{\partial z_1} = 0$ is the same as before: $z_1 = (1 - \frac{\gamma}{2})m$. The first order condition $\frac{\partial F}{\partial m} = 0$ gives $m - z_1 + (1-\gamma)(z_L - z_1) = z_L - m$. Substituting the first equation into the second and simplifying yields $m = \frac{2\, z_L}{4-\gamma}$ and so $z_1 = \frac{(2-\gamma)z_L}{4-\gamma}$.

Case III. $z_1 < z_L < z_2$. Once again the first order condition $\frac{\partial F}{\partial z_1} = 0$ is the same as before: $z_1 = (1 - \frac{\gamma}{2})m$. Since $z_L < z_2$ condition $\frac{\partial F}{\partial z_2} = 0$ becomes $\int_m^{z_2} dx - \int_{z_2}^1 dx = -(1-\gamma)(1-m)$. Expanding, this becomes $(z_2 - m) - (1 - z_2) = -(1-\gamma)(1-m)$. Solving for z_2, we get $z_2 = (1 - \frac{\gamma}{2})m + \frac{\gamma}{2}$. It is useful to note that $z_2 = z_1 + \frac{\gamma}{2}$ and hence $\frac{z_1+z_2}{2} = z_1 + \frac{\gamma}{4}$. The condition $\frac{\partial F}{\partial m} = 0$ becomes $m - z_1 + (1-\gamma)(z_L - z_1) = z_2 - m + (1-\gamma)(z_2 - z_L)$. Solve for m: $m = \frac{z_1+z_2}{2} - (1-\gamma)\big(z_L - \frac{z_1+z_2}{2}\big)$. Next substitute for $\frac{z_1+z_2}{2}$ to get $m = z_1 + \frac{\gamma}{4} - (1-\gamma)\big(z_L - (z_1 + \frac{\gamma}{4})\big) = (2-\gamma)(z_1 + \frac{\gamma}{4}) - (1-\gamma)z_L$. Finally substitute for z_1 to get $m = (2-\gamma)((1-\frac{\gamma}{2})m + \frac{\gamma}{4}) - (1-\gamma)z_L$. Solving for m will produce $m = \dfrac{\gamma^2 - 2\gamma + 4(1-\gamma)z_L}{2(\gamma^2 - 4\gamma + 2)}$.

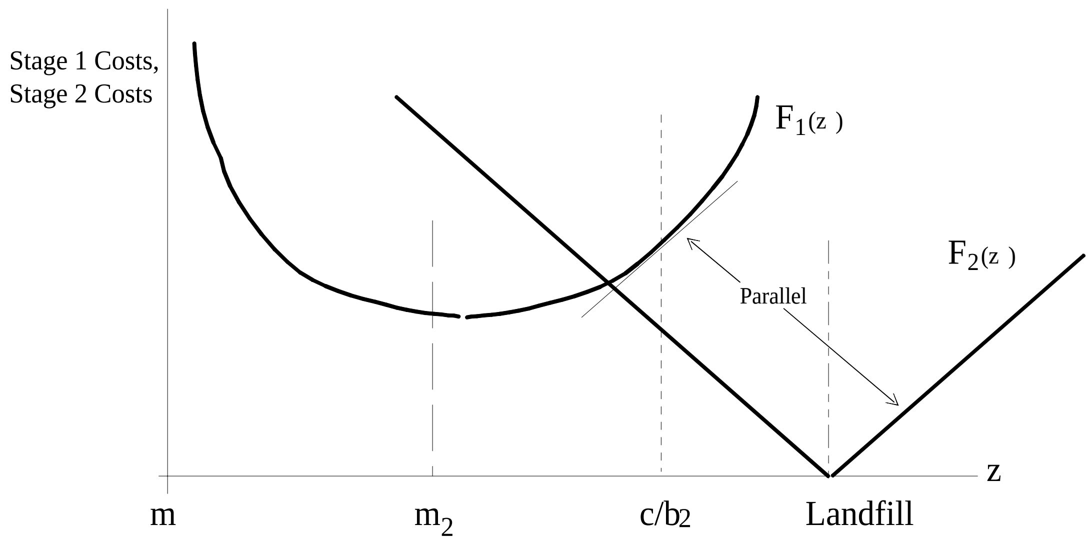

FIGURE 1. Decomposition of Transportation Costs:
Critical Point Case

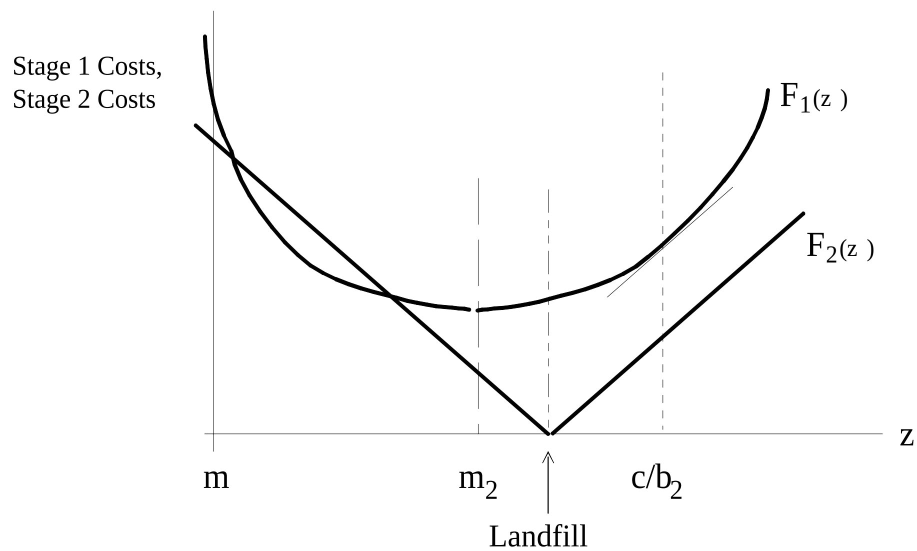

FIGURE 2. Decomposition of Transportation Costs:
Recycling Center at Landfill

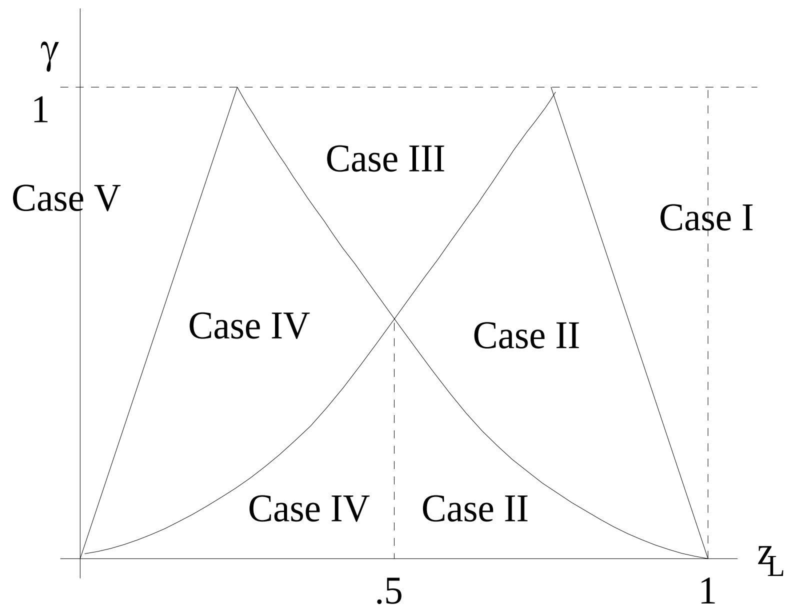

FIGURE 3. Parameters Diagram for the Uniform Distribution