Existence of positive solutions of a class of semilinear elliptic systems

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Appeared in *Journal of Natural Sciences I: Mathematics, Sichuan, 11 (1989), 100-110.*

Abstract

We studied some semilinear elliptic systems with power growth:

$$-\Delta u_1 = p_1(x)u_1^{\alpha_1}u_2^{\beta_1},\ -\Delta u_2 = p_2(x)u_1^{\alpha_2}u_2^{\beta_2}\ on\ \Omega;\ u_1 = u_2 = 0\ on\ \partial\Omega, \tag{0.1}$$

with $p_i,\ \alpha_i,\ \beta_i \geq 0$, where Ω is an exterior domain in R^n ($i.e.$, $0 \notin \Omega$) and $\Omega \supset G_a \equiv \{x \in R^n : |x| > a\}$ for some $a > 0$. Let $q_i(r) = \sup_{|x|=r} p_i(x)$ for $i = 1, 2$, then we showed (a) If $\alpha_1,\ \beta_2 \geq 1$, and for $i = 1, 2$, $\alpha_i + \beta_i > 1$ and $\int_{a_0}^{\infty} rq_i(r)dr < \infty$, then (1) has infinitely many nonnegative solutions that are positive on G_a. (b) If $\alpha_1 \geq 1$, $\alpha_1 + \beta_1 > 1$ and for $i = 1, 2$, $\int_{a_0}^{\infty} r^{\sigma_i}q_i(r)dr < \infty$, where $\sigma_1 = 1 - (n-2)\beta_1$, $\sigma_2 = n - 1 - (n-2)\beta_2$, then for any $A > a$, (1) has a nonnegative solution $u = (u_1, u_2)$ so that u_1 is positive on G_a and u_2 is positive on $G_a \backslash G_A$.

Key Words: *Semilinear elliptic systems, positive multiple solutions, lower-upper solution theorem, fixed point theorem.*