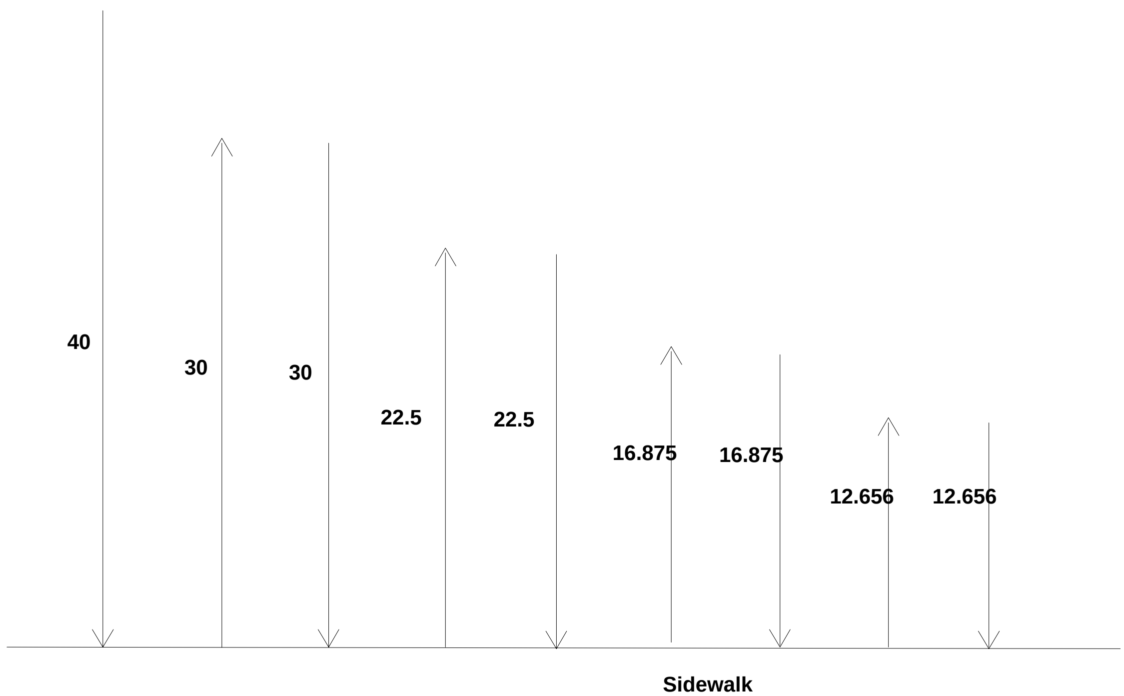
This is how the ball bounces!

A super ball is dropped from a window that's 40 feet up in the air. The ball bounces back 75% of the distance that it just fell. There's a way to compute the total distance that the ball has traveled. Of course, this is assuming that the ball always falls straight down and returns straight up. The theoretical is always prettier than the practical.



Consider just the distances going upward:

30, 22.5, 16.875, 12.65625, . . .

These are the terms of an infinite geometric sequence. This type of sequence (geometric) is formed

by taking that first term, the 30, and multiplying it by .75. $30 \times .75 = 22.5$. That second term is then multiplied by .75 to get the third term, and so on. The proper notation for this is:

$a_n = .75a_{n-1}$ The n^{th} term is equal to .75 times the term before it.

A wonderful thing about these infinite geometric sequences is that you can compute the sum of any number of terms in them. The formula for computing the sum of *n* terms is:

Sum $= \frac{a(1-r^n)}{1-r}$

The *a* is the first term. The *r* is the ratio or multiplier of the terms. And *n* is how many terms are being added up.

So, if you want to add up the first four terms of this sequence, $30 + 22.5 + 16.875 + 12.65625$, you can get out a calculator and get 82.03125 or you can use the formula.

Sum of first four terms $= \frac{30(1-.75^4)}{1-.75} = \frac{30(1-.31640625)}{.25} = \frac{30(.68359375)}{.25} = 82.03125$

With short lists, it's probably easier to just add them up with a calculator.

Another great feature of infinite geometric sequences is that if the ratio is between 0 and 1, the sequence "converges" to 0. The terms become so very small, that they're essentially equal to 0. This is especially nice when you want to add up all or lots of the terms. There's a formula for the sum of **all** of the terms of a converging geometric sequence. That formula is:

Sum $= \frac{a}{1-r}$ This formula is even easier than the one used for a specified number of terms. The sum of **all** of the terms of the sequence I've been using, if I never stopped multiplying by .75, would be:

Sum $= \frac{30}{1-.75} = \frac{30}{.25} = 120$ All of the terms in the sequence, going on forever and ever, will not exceed 120.

In *Algebra for Dummies*, on page 60, I've offered a "formula" for the total distance traveled by that bouncing ball. Where did I get this formula? Here goes.

Look at the "drawing" at the top. The ball first travels 40 feet down. This number isn't repeated, so my formula starts with the 40. Then the 30, 22.5 and so on are repeated, so I need 2 of each of these. Using the formula for the sum of a certain number of terms:

Sum $= \frac{a(1-r^n)}{1-r}$

$$40+2\left[\frac{30(1-.75^n)}{1-.75}\right] = 40+2\left[\frac{30(1-.75^n)}{.25}\right] = 40+2\left[\frac{30}{.25}\bullet\frac{(1-.75^n)}{1}\right]$$

$$= 40+2[120\bullet(1-.75^n)] = 40+240(1-.75^n)$$

So, the formula wasn't magically created. It has good mathematics behind it, but more than I wanted to cover in this book.